June 2019

CCSSO
Council of Chief State School Officers

Performance Level Descriptors (PLD) for Grades K-2 Mathematics

THE COUNCIL OF CHIEF STATE SCHOOL OFFICERS

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Performance Level Descriptors (PLD) for Grades K-2 Mathematics

We are grateful to our partners at **Student Achievement Partners** for their help in developing this guide.

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PERFORMANCE LEVEL DESCRIPTORS (PLD) FOR GRADES K-2 MATHEMATICS

BACKGROUND

The Performance Level Descriptor (PLD) document was created to provide details on the content progressions of the mathematics areas of focus at the kindergarten through grade 2 levels. The performance level descriptors in the table below describe the evidence needed to identify student proficiency along a progression of learning within each domain. Therefore, when tasks are given to students, these performance level descriptors will enable teachers to make instructional decisions to help move their students' learning forward.

STANDARDS FOR MATHEMATICAL PRACTICES

The descriptors below describe what teachers might observe related to the Standards for Mathematical Practice (SMPs) in observing students at different levels of proficiency. The descriptors serve as an important reminder that we are working to help all students progress toward meeting the goals outlined in the SMPs as appropriate for the K-2 grade band. Therefore, you will notice that level 1 is often described as "with support" or "with prompting" to recognize that we are supporting students as they work toward the goals outlined in level 2. Level 3 either ramps up the expectations of the SMPs to include explanations or sometimes has students exercising greater levels of autonomy and independence in demonstrating the SMPs.

Make sense of problems and persevere in solving them		
K-2 Students at Level 1:	**K-2 Students at Level 2:**	**K-2 Students at Level 3:**
• With support, represent problems using equations, verbal descriptions, and pictures to make sense of and solve problems.	• Independently use concrete objects or draw pictures to help conceptualize and solve problems. • Explain correspondences between equations, verbal descriptions, and pictures to make sense of and solve problems. • Recognize that problem solving takes time, and demonstrate perseverance in the face of initial challenge.	• Independently or with prompting, generate and solve new problems related to a given problem. • Explain why their answers make sense.
Reason abstractly and quantitatively		
K-2 Students at Level 1:	**K-2 Students at Level 2:**	**K-2 Students at Level 3:**
• Make sense of quantities and their relationships in familiar problems with small quantities.	• Make sense of quantities and their relationships in problem situations. • Attend to the meaning of quantities, not just how to compute them.	• Use different strategies and/or operations to show their solution(s) are correct.
Construct viable arguments and critique the reasoning of others		
K-2 Students at Level 1:	**K-2 Students at Level 2:**	**K-2 Students at Level 3:**
• With prompting, construct arguments using concrete referents such as objects, drawings, and actions. • Asks generic questions about another student's work but often without a level of specificity for the particular argument (e.g., How did you get that?).	• Construct arguments using concrete referents such as objects, drawings, and actions. • Listen to or read the arguments and explanations of others, decide whether they make sense, and ask clarifying questions.	• Incorporate feedback of others to improve a mathematical explanation or argument.
Model with mathematics		
K-2 Students at Level 1:	**K-2 Students at Level 2:**	**K-2 Students at Level 3:**
• With support, apply the mathematics they know to solve problems arising in everyday life and society (at level 1, these problems use small quantities for the grade level).	• Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.	• Routinely interpret their results, connecting quantities and solutions back to the context of the original problem.

Use appropriate tools strategically		
K-2 Students at Level 1: • Select and use appropriate mathematical tools when offered a limited set from which to choose. • Use relevant external mathematical resources with specific direction to do so.	**K-2 Students at Level 2:** • Consider the available tools when solving a mathematical problem. • Demonstrate familiarity with tools appropriate for their grade to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. • With support and guidance, identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems.	**K-2 Students at Level 3:** • Independently ask for tools to solve problems, especially when particular tools are not already in their reach. • Independently identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems.
Attend to precision		
K-2 Students at Level 1: • With support, try to communicate precisely to others. • With support, gradually refine informal language to more formal language, such as using clear definitions in discussion. • With support, such as sentence frames or sentence starters, try to formulate explanations.	**K-2 Students at Level 2:** • Try to communicate precisely to others. • Try to use clear definitions in discussion with others and in their own reasoning. • Give explanations to each other that sometimes prompt questions for further clarification to refine the explanation.	**K-2 Students at Level 3:** • Communicate precisely to others. • Use clear definitions in discussion with others and in their reasoning. • Give carefully formulated explanations to each other.
Look for and make use of structure		
K-2 Students at Level 1: • With support, use manipulatives or concrete representations to see structures or patterns (e.g., students may use a 10 frame to see that nine dots and one more dot make 10).	**K-2 Students at Level 2:** • Look closely to discern a pattern or structure (e.g., notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have).	**K-2 Students at Level 3:** • Notice patterns and structures within a problem and generalize this and apply it to other problems with similar structure. • Communicate the underlying mathematical understanding related to the pattern or structure identified.
Look for and express regularity in repeated reasoning		
K-2 Students at Level 1: • With prompting, notice that calculations are repeated, and identify general methods and/or shortcuts.	**K-2 Students at Level 2:** • Notice if calculations are repeated, and look for both general methods and/or shortcuts.	**K-2 Students at Level 3:** • Notice if calculations are repeated, and look for both general methods and/or shortcuts, and generate additional examples to demonstrate the adequacy of the general method or shortcut.

Counting and Cardinality		
Kindergarten	**Grade 1**	**Grade 2**
Students use numbers, including written numerals, to represent quantities, counting out a given number of objects and comparing sets or numerals.	(NOTE: CCSS does not identify a critical area in grade 1 and 2 for counting and cardinality concepts.)	(NOTE: CCSS does not identify a critical area in grade 1 and 2 for counting and cardinality concepts.)
Standards: K.CC.A.1, K.CC.A.2, K.CC.A.3, K.CC.A.1, K.CC.A.1, K.CC.B.4, K.CC.B.5, K.CC.C.6, K.CC.C.7		
The Performance Descriptors provide brief examples of what students at each grade level should know, understand, and be able to do. Teachers can use the different Performance Descriptor levels to plan instruction and identify opportunities to formatively assess student understanding. **Level 1** (below proficient)—Students performing at level 1 inconsistently demonstrate their understanding and show a partial command of the standards. **Level 2** (proficient)—Students performing at level 2 consistently demonstrate understanding and show a command of the standards. Level 2 (proficient) is the minimum goal for all students. **Level 3** (above proficient)—Students performing at level 3 demonstrate a deeper understanding and command of the standards by making connections across grade-level standards and domains.		
Kindergarten students at level 3: • Know the number just after or just before a given number. • Organize large sets of objects into groups as a method of simplifying counting. • In making comparisons, use more formal mathematical language, such as "greater than," "less than," or "equal to." • Justify comparisons between numbers orally or in writing.		

Kindergarten students at level 2: • Count to 100 by ones and by tens. • Count forward beginning from a given number other than 1. • Write numbers from 0 to 20. • Represent a number of objects with a written numeral 0 to 20. • Understand the relationship between numbers and quantities. • When counting objects, say the number names in the standard order, pairing each object with one number (one-to-one correspondence). • Understand that the last number name stated tells the number of objects counted (cardinality). • Understand that each successive number refers to a quantity that is one larger. • Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration. • Count out objects to represent a given number from 1 to 20. • Know the number of objects is the same regardless of their arrangement or the order in which they were counted. • Recognize small quantities 1 to 5 without counting (subitize). • Compare two numbers between 1 and 10 presented as written numerals. • Determine whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group.		

Kindergarten students at level 1: • Count by ones to a number less than 100, or count to 100 with errors/ omissions. • Count by tens from 0 to 100 with support, or count by tens to 100 with errors/omissions. • Count forward from a given number within 10. • Given a set of objects from 0 to 10, count in the standard order and correctly state the total counted. • Count out objects to represent a given number from 0 to 10. • Write some numbers from 0 to 20 with support. • Compare two numbers between 1 and 10 presented as written numerals with support, such as counting out groups of physical objects with one-to-one correspondence or by referencing the counting sequence. • For easier comparisons (e.g., a group of seven objects compared to one object), determine whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group.		

Numbers and Operations in Base Ten		
Kindergarten	**Grade 1**	**Grade 2**
The foundational understanding of the base ten system occurs in kindergarten. The Numbers and Operations in Base Ten domain consists of the major work of the grade level. Place-value understanding provides the foundation for students to extend their understanding of whole numbers and decimals in later grade levels. The content should build across the kindergarten school year and should be closely linked with the Counting and Cardinality and Operations and Algebraic Thinking domains. Once students have an understanding of counting from 1 to 9, the focus turns to the number 10, where students begin to understand that 10 is ten ones and that ten can be decomposed into pairs, such as 1 + 9, 2 + 8, and so on. Placing the 10 ones into patterns is beneficial for students, such as placing the 10 objects into two groups of five. Students begin to connect this understanding to the numbers 11 through 19 and their meanings; for example, through composing and decomposing, students begin to think of 12 as 10 ones and two more. This understanding is students' first experience with the structure of mathematics. Mathematics content knowledge and conceptual understanding will be developed as students use a number of different concrete materials for modeling purposes, such as five and ten frames and linking cubes. Other representatives will be used to explain student thinking, such as drawings and numerals.	Building on kindergarten content, first-graders begin to see 10 as a unit of 10 ones. A foundational understanding of place value is even more critical as students move to understand addition and subtraction beyond basic facts. The structure of math is continued through whole numbers. Whole numbers are thought of in terms of place value, where the number in the tens place represents that many groups of 10, and the number in the ones place represents that many ones. Once students understand the concept of teen numbers being made up of one ten and some ones, students can begin developing their understanding of groups of 10 with no ones left over: 20, 30, 40, and so on. The next step is for students to begin working with other numbers and describing them based on place value (e.g., 23 is 2 tens and 3 ones, or 23 ones). A variety of mathematical representations will continue to be used to build conceptual understanding. Concrete materials should be connected to student drawings and symbolic notation, including <, >, and =; first-graders recognize that the digit in the tens place is important in determining the size of a two-digit number, which allows them to compare two two-digit numbers. By using a variety of representations and developing strategies, students will be able to compute with understanding sums to 100. Understanding of subtraction with two-digit numbers will only include differences of multiples of 10 at this grade level.	Conceptual understanding developed in kindergarten and grade 1 should be connected to the major tasks of extension of place value, counting, and comparing numerals to 1,000. Place-value understanding is extended with the concept of building 100 out of 10 groups of 10 and later 1,000 out of 10 groups of 100. Students make the connection that this repeated reasoning is the basis of the place-value system. With the ability to compose and decompose groups of 10, students begin to work on two-digit addition and subtraction, becoming fluent as their understanding of place value and properties broadens. As students receive scaffolding from concrete to pictorial, and then to abstract representations, they will be able to add and subtract to 1,000. Number sense and mental math can be applied to addition and subtraction work; students can also utilize the relationship between addition and subtraction to problem solve. Students can begin to add and subtract three-digit numbers through the use of a variety of representations. Students are expected to explain their reasoning based on place value and strategies. Second-graders are not required to use the standard algorithm for addition and subtraction. Work should focus on developing and using effective strategies that work for all numbers so generalization can occur with numbers beyond 1,000 in later grades. Concrete representations are helpful for students to develop a conceptual understanding of regrouping.
Standards: K.NBT.1	**Standards:** 1.NBT.1, 1.NBT.2, 1.NBT.3, 1.NBT.4, 1.NBT.5, 1.NBT.6	**Standards:** 2.NBT.1, 2.NBT.2, 2.NBT.3, 2.NBT.4, 2.NBT.5, 2.NBT.6, 2.NBT.7, 2.NBT.8, 2.NBT.9

The Performance Descriptors provide brief examples of what students at each grade level should know, understand, and be able to do. Teachers can use the different Performance Descriptor levels to plan instruction and identify opportunities to formatively assess student understanding.

Level 1 (below proficient)—Students performing at level 1 inconsistently demonstrate their understanding and show a partial command of the standards.

Level 2 (proficient)—Students performing at level 2 consistently demonstrate understanding and show a command of the standards. Level 2 (proficient) is the minimum goal for all students.

Level 3 (above proficient)—Students performing at level 3 demonstrate a deeper understanding and command of the standards by making connections across grade-level standards and domains.

Kindergarten students at level 3:	**Grade 1 students at level 3:**	**Grade 2 students at level 3:**
• Compose and decompose numbers from 11 to 19 into 10 ones and some further ones (e.g., compose and decompose primarily through the use of equations showing different decompositions with minimal use of drawings, such as 18 = 10 + 5 + 3); understand that these numbers are composed of 10 ones and one, two, three, four, five, six, seven, eight, or nine ones.)	• Use place-value understanding to show different decompositions of the same number (e.g., 57 is 5 tens and 7 ones, 4 tens and 17 ones, or 57 ones). • Connect different strategies for solving the same problem, solving unknown addend/subtrahend or unknown digit(s) problems. • Use abstract and quantitative reasoning to determine if an answer makes sense by thinking about the value of numbers. • Use the structure of expressions and the meaning of equal and inequality signs to complete comparisons (e.g., 3 tens + 7 ones = ___ tens + 17 ones).	• Fluently add and subtract within 100 by selecting efficient strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. • Add and subtract within 1,000 using concrete models or drawings and selecting efficient strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. • Articulate connections between different strategies for the same problem. • Use place-value understanding to solve unknown addend/subtrahend or unknown digit(s) problems.

Kindergarten students at level 2:	Grade 1 students at level 2:	Grade 2 students at level 2:
• Compose and decompose numbers from 11 to 19 into 10 ones and some further ones (e.g., by using objects or drawings and recording each composition or decomposition by a drawing or equation, such as 18 = 10 + 8); understand that these numbers are composed of 10 ones and one, two, three, four, five, six, seven, eight, or nine ones).	• Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral. • Understand that the two digits of a two-digit number represent the amount of tens and ones. Understand the following as special cases: o 10 can be thought of as a bundle of 10 ones—called a "ten." o The numbers from 11 to 19 are composed of a 10 and one, two, three, four, five, six, seven, eight, or nine ones. o The numbers 10, 20, 30, 40, 50, 60, 70, 80, and 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). • Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <. • Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones, and sometimes it is necessary to compose a ten. • Given a two-digit number, mentally find 10 more or 10 less than the number without having to count; explain the reasoning used. • Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences) using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	• Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones (e.g., 706 equals 7 hundreds, 0 tens, and 6 ones). Understand the following as special cases: o 100 can be thought of as a bundle of ten tens—called a "hundred." o The numbers 100, 200, 300, 400, 500, 600, 700, 800, and 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). • Count within 1,000; skip-count by 5s, 10s, and 100s. • Read and write numbers to 1,000 using base-ten numerals, number names, and expanded form. • Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits using >, =, and < symbols to record the results of comparisons. • Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. • Add up to four two-digit numbers using strategies based on place value and properties of operations. • Add and subtract within 1,000 using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; sometimes it is necessary to compose or decompose tens or hundreds. • Mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100-900. • Explain why addition and subtraction strategies work using place value and the properties of operations.

Kindergarten students at level 1:	Grade 1 students at level 1:	Grade 2 students at level 1:
• With support, compose and decompose numbers from 11 to 19 into 10 ones and some further ones (e.g., by using objects or drawings to show that these numbers are composed of 10 ones and one, two, three, four, five, six, seven, eight, or nine ones). • Compose and decompose some numbers from 11 to 19 but cannot necessarily connect composition and decomposition to a written equation. Students may be able to use concrete or pictorial models to state compositions and decompositions but not necessarily connect them to a written equation that records the composition/decomposition.	• Count to 120 starting from some number less than 120. In this range, read and write some numerals with support, and represent a number of objects with a written numeral. • Understand that the two digits of a two-digit number represent amounts of tens and ones. Show some understanding of the following as special cases: o 10 can be thought of as a bundle of 10 ones—called a "ten." o The numbers from 11 to 19 are composed of a 10 and one, two, three, four, five, six, seven, eight, or nine ones. o The numbers 10, 20, 30, 40, 50, 60, 70, 80, and 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). • Verbally compare two two-digit numbers based on meanings of the tens and ones digits with "less than," "greater than," or "equal to." • With support, add within 100 using concrete models, drawings, or strategies based on place value. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; sometimes it is necessary to compose a ten. • Given a two-digit number, find 10 more or 10 less than the number, possibly still using counting instead of place-value understanding. • With support, subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences) using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	• With support, understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones (e.g., 706 equals 7 hundreds, 0 tens, and 6 ones). Understand the following as special cases: o 100 can be thought of as a bundle of 10 tens—called a "hundred." o The numbers 100, 200, 300, 400, 500, 600, 700, 800, and 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). • Count within 1,000 with errors. • Skip-count by 5s, 10s, and 100s with support. • Read and write numbers to 1,000 with some errors using one or more of these forms: base-ten numerals, number names, and expanded form. • With support, add and subtract within 1,000 using concrete models or drawings and strategies based on place value. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; sometimes it is necessary to compose or decompose tens or hundreds. • With support, add 10 or 100 to a given number 100-900, and subtract 10 or 100 from a given number 100-900.

Operations and Algebraic Thinking		
Kindergarten	**Grade 1**	**Grade 2**
Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects or eventually with equations such as 5 + 2 = 7 and 7 – 2 = 5. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.	Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths) to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding to get two is the same as counting to two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.	Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1,000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.
Standards: K.OA.1, K.OA.2, K.OA.3, K.OA.4, K.OA.5	**Standards:** 1.OA.1, 1.OA.2, 1.OA.3, 1.OA.4, 1.OA.5, 1.OA.6, 1.OA.7, 1.OA.8	**Standards:** 2.OA.1, 2.OA.2, 2.OA.3, 2.OA.4

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Level 3 (above proficient)—Students performing at level 3 demonstrate a deeper understanding and command of the standards by naturally making connections to the standards for mathematical practice.

Kindergarten students at level 3:	Grade 1 students at level 3:	Grade 2 students at level 3:
• Solve joining and separating situations with sets of objects, and consistently provide written equations to match their thinking. • Make conjectures by verbally assessing the reasonableness of answers when solving a real-world problem. They ask themselves, "Does this make sense?" • Connect mathematical models to real-world problems. • Construct viable arguments when using mathematical models (e.g., I used my hands, because one hand is 5, and I counted 2 more fingers) when solving word problems. • Explain addition and subtraction situations in authentic real-world experiences or across content areas. • Know from memory addition and subtraction facts within 5.	• Make conjectures and assess the reasonableness of answers when solving a real-world problem. • Connect mathematical models to real-world problems. • Critique the reasoning of others when solving word problems. • Use efficient strategies (based on place-value properties of operations, relationship of addition/subtraction) to illustrate their understanding and explain their thinking to others. • Explain addition and subtraction situations in authentic real-world experiences or across content areas. • Know from memory addition and subtraction facts within 10.	• Make conjectures and assess the reasonableness of answers when solving a real-world problem. • Connect mathematical models to real-world problems. • Critique the reasoning of others when solving word problems. • Solve words problems within 100; develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers. • Explain addition and subtraction situations in authentic real-world experiences or across content areas.

Kindergarten students at level 2:	Grade 1 students at level 2:	Grade 2 students at level 2:
• Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, and/or equations. • Solve addition and subtraction word problems, and add and subtract within 10 (e.g., by joining and separating objects or drawings to represent the problem). • Reason about and problem solve word problems within 10, and explain their reasoning. • Decompose numbers less than or equal to 10 into pairs in more than one way (e.g., by using objects or drawings), and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1). • For any number from 1 to 9, find the number that makes 10 when added to the given number (e.g., by using objects or drawings), and record the answer with a drawing or equation. • Fluently add and subtract within 5.	• Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions (e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem). • Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20. • Apply properties of operations as strategies to add and subtract. • Understand subtraction as an unknown-addend problem. • Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). • Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 - 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). • Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. • Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. • Reason and problem solve word problems within 20 and explain their reasoning.	• Reason about and problem solve one- and two-step word problems within 100. • Solve add-to, take-from, put-together, take-apart, and comparison types of story problems where the unknown could be in the result, the change, or the start. • Add up to five one-digit numbers arranged in a rectangular array using strategies based on structure of numbers, properties, and place value. • Understand even and odd, and use the understanding of this structure to count and solve problems. • Fluently add and subtract within 20 using mental strategies including problems with missing addends and subtrahends. • Know from memory all sums of two one-digit numbers.

Kindergarten students at level 1:	Grade 1 students at level 1:	Grade 2 students at level 1:
• Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, and/or verbal explanations with support. • Model a simple joining and separating story problem with support. • Understand the concept of putting together and taking apart within 10 with support. • Add and subtract within 5 with support.	• Model add-to, take-from, put-together, take-apart, and compare situations within 20 with support (e.g., provide specific models and/or manipulatives for the student to utilize). • Apply given strategies, properties, and/or relationships to add and subtract within 20. • Understand that expressions can have the same value but may not use formal mathematical notation to describe the equality (i.e., connecting them with an equal sign). • Determine an unknown in the sum or difference position.	• Model one-step add-to, take-from, put-together, take-apart, and compare situations within 100 with support (e.g., provide specific models and/or manipulatives for the student to utilize). • Fluently add and subtract sums and differences within 20 with unknown sums or differences. • Add up to five one-digit numbers arranged in a rectangular array using concrete models.

Measurement and Data		
Kindergarten	**Grade 1**	**Grade 2**
Students identify and name measurable attributes of objects, such as length or weight. They are able to describe several measurable attributes of a single object. They directly compare two objects with a common measurable attribute to see which object has more/less of the attribute described. They can also describe the difference. Students classify objects into categories, count the number of objects in each category, and arrange the categories by the number of objects in each set (greatest to least or least to greatest). Vocabulary development is a large part of the development of these standards that describe measurable attributes and comparative language.	Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement. (If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C.) A child with this understanding can use a third object to compare the length of the other two objects. This principle applies to measurement of other quantities as well. (CCSS, page 13)	Students recognize the need for standard units of measure (centimeter and inch), and they use rulers and other measurement tools with the understanding that linear measurement involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length. Students apply their understanding of measurement to solving addition and subtraction problems involving lengths. They use drawings and diagrams (number line, etc.), paper and pencil, concrete models, and equations to solve measurement word problems. Students also apply their understanding of measurement to generate measurement data to create a line plot and create picture and bar graphs to represent data and solve word problems based on the data. Grade 2 students also work with time and money.
Standards: K.MD.A.1, K.MD.A.2, K.MD.B.3	**Standards:** 1.MD.A.1, 1.MD.A.2	**Standards:** 2.MD.A.1, 2.MD.A.2, 2.MD.A.3, 2.MD.A.4, 2.MD.B.5, 2.MD.B.6

The Performance Descriptors provide brief examples of what students at each grade level should know, understand, and be able to do. Teachers can use the different Performance Descriptor levels to plan instruction and identify opportunities to formatively assess student understanding.

Level 1 (below proficient)—Students performing at level 1 inconsistently demonstrate their understanding and show a partial command of the standards.

Level 2 (proficient)—Students performing at level 2 consistently demonstrate understanding and show a command of the standards. Level 2 (proficient) is the minimum goal for all students.

Level 3 (above proficient)—Students performing at level 3 demonstrate a deeper understanding and command of the standards by making connections across grade-level standards and domains.

Kindergarten students at level 3:	Grade 1 students at level 3:	Grade 2 students at level 3:
Data: • Can sort objects with multiple attributes. • Can sort objects in multiple ways based on different attributes. • Describe similarities and differences between category sizes. **Measurement:** • Order three objects by a measurable attribute (e.g., biggest to smallest, etc.). • Compare a measurable attribute of one object with multiple objects (e.g., I am taller than my baby sister and shorter than the treehouse). • Use measurement data as a justification for scientific explorations (e.g., the plant in the window is taller than the plant in the closet because it received sun and water).	**Data:** • Select a representation of data that supports one or more questions being explored and explains how the representation can be used to effectively respond to the question(s). **Measurement:** • Relate the measurement of time to real-world situations. • Justify indirect comparisons by using length measurements (e.g., I claim that if my pencil is shorter than the book, and the book is shorter than the table, then my pencil is shorter than the table. My evidence is that my pencil is 6 paper clips long, the book is 25 paper clips long, and the table is 75 paper clips long. So, I am correct because 6 is less than 75.). • Use measurement data as a justification for scientific explorations (e.g., the car rolled the length of 3 pieces of paper).	**Data:** • Select a representation of data (bar graph, picture graph, line plot, or other display) that supports one or more questions being explored and explains how the representation can be used to effectively respond to the question(s). **Measurement:** • Use measurement data as a justification for scientific explorations (e.g., the paper airplane made out of newspaper flew 5 yards longer than the paper airplane made out of construction paper).

Kindergarten students at level 2:	Grade 1 students at level 2:	Grade 2 students at level 2:
Data: • Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. **Measurement:** • Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object. • Directly compare two objects with a measurable attribute in common to see which object has more of/less of the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*	**Data:** • Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another. **Measurement:** • Order three objects by length; compare the lengths of two objects indirectly by using a third object. • Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.* • Tell and write time in hours and half-hours using analog and digital clocks.	**Data:** • Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph. • Generate measurement data by measuring lengths of several objects to the nearest whole unit. Show the measurements by making a line plot where the horizontal scale is marked off in whole-number units. **Measurement:** • Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. • Measure the length of an object twice using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. • Estimate lengths using units of inches, feet, centimeters, and meters. • Measure to determine how much longer one object is than another expressing the length difference in terms of a standard-length unit. • Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units (e.g., by using drawings, such as drawings of rulers, and equations with a symbol for the unknown number to represent the problem). • Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number-line diagram. • Tell and write time from analog and digital clocks to the nearest five minutes using a.m. and p.m. • Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately (e.g., If you have 2 dimes and 3 pennies, how many cents do you have?).

Kindergarten students at level 1:	Grade 1 students at level 1:	Grade 2 students at level 1:
Data: • Identify objects with a given attribute (e.g., point to the green objects). • (See CC domain for additional counting and ordering support.) **Measurement:** • Identify length/weight/height as an attribute. • Directly compare length using real-life situations (e.g., people's height).	**Data:** • Organize and represent simple data (e.g., organizing by color) with up to three categories with support. • (See OA and NBT domain for additional support.) **Measurement:** • Order three objects by a measurable attribute (e.g., biggest to smallest, etc.). • Compare a measurable attribute of one object with multiple objects (e.g., I am taller than my baby sister and shorter than the treehouse.). • Identify how to correctly measure an object within a given model. • Understand that time is measured by clocks.	**Data:** • Connect the data to the parts of a graph or line plot (i.e., title, labels, key, what the X on a line plot signifies). • (See OA and NBT domain for additional support.) **Measurement:** • Identify coins and know their value. • Identify length tools such as rulers, yardsticks, meter sticks, and measuring tapes. • Understand half of a length unit. • Understand number lines do not have to start at zero. • (See OA and NBT domain for additional support.)

Geometry		
Kindergarten	**Grade 1**	**Grade 2**
Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations) as well as three-dimensional shapes, such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.	Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.	Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.
Standards: K.G.1, K.G.2, K.G.3, K.G.4, K.G.5, K.G.6	**Standards:** 1.G.1, 1.G.2, 1.G.3	**Standards:** 2.G.1, 2.G.2, 2.G.3
The Performance Descriptors provide brief examples of what students at each grade level should know, understand, and be able to do. Teachers can use the different Performance Descriptor levels to plan instruction and identify opportunities to formatively assess student understanding. **Level 1** (below proficient)—Students performing at level 1 inconsistently demonstrate their understanding and show a partial command of the standards. **Level 2** (proficient)—Students performing at level 2 consistently demonstrate understanding and show a command of the standards. Level 2 (proficient) is the minimum goal for all students. **Level 3** (above proficient)—Students performing at level 3 demonstrate a deeper understanding and command of the standards by making connections across grade-level standards and domains.		
Kindergarten students at level 3: • Describe two-dimensional and three-dimensional shapes using more formal mathematical vocabulary.	**Grade 1 students at level 3:** • Include formal mathematical vocabulary in describing the attributes of shapes. • Explain why a shape belongs to a particular category by referencing its attributes as they relate to the definition.	**Grade 2 students at level 3:** • Justify that a shape has been equally partitioned.

Kindergarten students at level 2:	Grade 1 students at level 2:	Grade 2 students at level 2:
• Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as "above," "below," "beside," "in front of," "behind," and "next to." • Correctly name shapes regardless of their orientations or overall size. • Identify shapes as two-dimensional (lying in a plane, "flat") or three dimensional ("solid"). • Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners"), and other attributes (e.g., having sides of equal length). • Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes. • Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*	• Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes. • Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. • Partition circles and rectangles into two and four equal shares, describe the shares using the words "halves," "fourths," and "quarters," and use the phrases "half of," "fourth of," and "quarter of." Describe the whole as two of or four of the shares. For these examples, understand that decomposing into more equal shares creates smaller shares.	• Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. • Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. • Partition a rectangle into rows and columns of same-size squares, and count to find the total number of them. • Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words "halves," "thirds," "half of," "a third of," etc., and describe the whole as two halves, three thirds, or four fourths. Recognize that equal shares of identical wholes need not have the same shape.
Kindergarten students at level 1:	**Grade 1 students at level 1:**	**Grade 2 students at level 1:**
• Locate or describe objects in the environment using names of shapes but only for a subset of shapes in a particular category (e.g., can identify certain triangular objects that match the student's schema). • Correctly name shapes in common or familiar orientations with common or familiar characteristics (e.g., recognizes rectangles that match the student's schema but not all that meet the definition). • Use informal language to describe attributes of two- and three-dimensional shapes. • Draw or model simple two-dimensional composite shapes in the world.	• Create composite shapes using two- and three-dimensional shapes. • Identify some defining attributes (e.g., recognizing a shape with three sides as a triangle but not knowing that it must also be closed) and some non-defining attributes (e.g., recognizing color as non-defining but not orientation) of two-dimensional shapes. • Partition a rectangle into equal shares. • Recognize equal shares of partitioned shapes. • Count the number of equal shares of partitioned shapes.	• Recognize and name shapes based on specific attributes (e.g., circles, triangles, quadrilaterals, pentagons, hexagons, and cubes) when presented visually. • Identify partitioned shares of circles and/or rectangles as halves or fourths/quarters. • Partition circles and rectangles into halves and fourths.

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